GRANT /TR/IN/15
2002 087 645

585945
43p.

Statistical Methods for Rapid Aerothermal Analysis and Design Technology

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January 31, 2002

Abstract

The cost and safety goals for NASA's next generation of reusable launch vehicle (RLV) will require that rapid high-fidelity aerothermodynamic design tools be used early in the design cycle. To meet these requirements, it is desirable to establish statistical models that quantify and improve the accuracy, extend the applicability, and enable combined analyses using existing prediction tools. The research work was focused on establishing the suitable mathematical/statistical models for these purposes. It is anticipated that the resulting models can be incorporated into a software tool to provide rapid, variable-fidelity, aerothermal environments to predict heating along an arbitrary trajectory. This work will support development of an integrated design tool to perform automated thermal protection system (TPS) sizing and material selection.

Introduction

Recent design experience with NASA's X-37 has demonstrated the need for considering higher fidelity aerodynamic heating early in the design cycle. In the case of X-37, the vehicle shape was optimized for aerodynamic performance and resulted in severe aerodynamic heating that forced costly redesign of the nose and wing surfaces and lowered flight margins. The availability of higher fidelity aerothermal analysis earlier in the design cycle could have prevented these problems.

Development of this technology impacts NASA's goal of reduced cost by enabling faster and more optimized design cycles. Utilizing higher fidelity analyses earlier in the design will avoid the delay and expense of late design changes. Optimizing the thermal-structural and TPS design in the process will minimize vehicle weight leading to lower launch cost. The technology described is applicable across all next generation systems/architectures for any vehicle configuration and includes metallic and ceramic TPS and hot structures.

There will be two phases to this effort and they are model development and model validation. The initial phase will be to explore and/or develop the statistical/mathematical methods that can be used to transform the point wise aeroheating predictions of current tools to yield complete aerothermal environments through a trajectory corridor. The approach is intended to identify statistical/math models that best characterize and/or model a set of sphere stagnation data. Once several acceptable models have been identified, they will be tested in the second phase to see if they are able to predict heating values within the normal trajectory of corridor values.

Description of Sphere Stagnation Data Sets

There were two worksheets for analysis; one containing the full trajectory space (1269 measurements) and the second with measurements only within the entry corridor (138 measurements). For these data sets, a sphere shape configuration is used and the measurements are taken at the stagnation point on the sphere. For simplicity, a sphere of one foot radius is assumed. Generally, as the sphere radius decreases, the heat rate measurements will increase.

For each case, eleven (11) variables are labeled on the worksheets. The first three variables (altitude, velocity, and wall temperature) are considered the basic independent variables. The other variables are derived from these basic three variables. For example, the density, pressure, and temperature are direct functions of altitude. The Mach number, dynamic pressure, Reynolds number, and energy variables are combinations of the 3 basic variables, e.g. dynamic pressure equals density*velocity*velocity. The response variable of interest is heat rate.

The heating rate values in the spreadsheet were generated using MINIVER-tape calculations. The datasets contain values for velocity and altitude but no angle-of-attack values because of the sphere assumption.

Analysis Methods

Our goal is to begin with some rudimentary analysis on these type datasets to explore the behavior and relationships, correlations, summary statistics, graphics including contour plots and 3-D plots, robust and residual analyses. Several regression models were investigated including multiple linear regression, polynomial regression, step-wise, best subset, quadratic and cubic regression models as well as some nonlinear regression models. Regression analysis allows one to model the relationship between a response variable and one or more predictor variables. One of the useful features of a regression model is that it can be used to predict or estimate a future response value based on a given set of values of the predictor variables.

Regression analysis results usually include the following: regression equation, predictor table, summary statistics, ANOVA Table, list of unusual observations, contour plots, 3-D plots and residual plots. To appropriately use the t-test, F-test and associated confidence intervals, the data are assumed to meet certain conditions. These include (1) the residuals (error component) are assumed to be normally distributed, (2) variation is constant (homoscedasity) and (3) independence. The study of unusual patterns in the residuals through residual analysis may indicate underlying weaknesses in the model.

Data Exploration

Table 1 in the Appendix lists the 138 data cases used to develop a predictive model relating the response variable, heat rate, to the 10 predictor variables shown. Due to the large quantity of data, the full trajectory data set will note be included in the appendix. It should be noted that unless otherwise indicated tables and figures will appear in the Appendix. The summary statistics for the independent and response variables are provided in Tables 2 and 2a.

Prior to the model building activities, graphical methods were used to help identify any underlying relationships between the variables being studied. Matrix plots of cross-graphs of the variables are provided in Tables 3 and 3a. The plots highlight

- Quadratic relationship between heat rate and altitude, temp
- Strong positive linear relationship between mach and velocity, mach and altitude, velocity and altitude
- Exponential relationship between heat rate and Reynolds number.

On the other hand, Tables 4 and 4a consist of the Correlation Matrix that specifies the Pearson correlation and corresponding p-values. Given the inherent relationships between the independent or predictor variables, a principal components analysis was performed to help define a set of orthogonal variables so that the first principal component accounts for the largest possible amount of the total dispersion in the data, the second principal component accounts for the second largest possible amount of the total dispersion in the data, etc. This would be beneficial in helping to identify a subset list of candidate predictor variables for the analysis. Tables 5a and 5b show the results of the analysis using the correlation matrix of the predictor variables as input to the principal components analysis.
Another method that was used to identify a subset list of candidate predictor variables is best subset selection. Table 6 shows the results of this analysis for the corridor data set. Best subset selection identified altitude, velocity, mach, dynamic pressure and Reynolds as the top five prediction variables.

Classification and Regression Tree (CART) based models are exploratory techniques for uncovering structure in data that are used for:

- developing prediction rules that can be rapidly evaluated
- screening variables
- assessing the adequacy of linear models
- summarizing large data sets for both classification and regression problems.

Tables 7a and 7b show the results of the CART analysis and Figure 1 shows the resulting CART tree. CART selected velocity, mach, altitude, energy and dynamic pressure as the primary prediction variables. The tree indicates that for those data cases with velocity measurements less than 13750, the predicted values for heat rate are generally below thirty. Furthermore, if values of mach are below 7.7, then the predicted heat rate is approximately 3.909.

The classical regression methods are often used to obtain models for prediction. The challenge is the development of the best mathematical expression to describe in some sense the behavior of a random variable of interest as a function of one or more independent or predictor variables. The classical regression techniques however make several strong assumptions about the underlying data, and the data can fail to satisfy these assumptions in several different ways as indicated the Analysis Methods section.

In the case where there are one or more outliers in the data or the data may not be fitted well by any straight line, robust regression methods come into play. These methods minimize the effect of the outliers and can be useful in helping to identify the outliers in the data.

Scatterplot smoothers are useful tools for fitting arbitrary smooth functions to a scatter plot of data points. The smoother summarizes the trend of the response as function of the predictor variables. All of the above analysis methods are used to explore the given data sets.

Results

Several approaches were used in the exploration and identification of statistical/mathematical models for the given data sets including the classical multiple regression, classification and regression trees (CART), and other advanced analysis methods. One of the interesting results concerns a comparison of some initial multiple regression model types using only the independent predictor variables for both full and corridor data sets. These results are summarized below in Table A that includes Data, Coefficient of Determination R^2, Adjusted R^2, fit standard error S, F-statistics and the model type.

Table A: Comparison of Model Types for Full and Corridor Data using Three (3) Independent Predictor Variables

Data	R^2	Adj R^2	Std Error	F-statistic	Model Type
Full	48.3%	48.2%	196.5	394.15	Linear
Corridor	85.2%	84.9%	8.72	257.21	Linear
Full	84.4%	84.4%	108.0	1140.45	Quadratic
Corridor	97.6%	97.5%	3.54	895.68	Quadratic
Full	85.6%	85.5%	103.9	937.21	Cubic, Interact
Corridor	99.7%	99.6%	1.34	4751.89	Cubic, Interact

In reviewing Table A, an obvious conclusion is that the regression models appear to be more appropriate for the corridor data than the full trajectory data for all model types.

For every model type comparison in Table A, there are higher R2 and adjusted R2 values and lower standard error values for the corridor data set. A more in-depth analysis was conducted on the corridor data set as it more realistically simulates possible entry trajectory and heating rates of an experimental space vehicle. The structured approach that was used for model identification consists of the following: (1) analyze the summary statistics results for errors and consistency, (2) conduct preliminary analysis using only the independent predictor variables, (3) use graphical methods to identify underlying relationships, (4) employ methods (Best Subset Selection, Principal Components, etc..) that aid in identifying most likely additional predictor variables and (5) specify a model using classical regression, classification and regression trees (CART) and other advanced statistical methods. The results are summarized in Table B below that includes rank, (R^2), adjusted R^2, fit standard error (S), F-statistics and the model type. Additional analysis and results on the number 1, number 2 and number 6 ranked models are provided in the Appendix in Tables 8 to 16 and Figures 2 to 12. Regression model ranked 6 has all predictor variables except altitude, velocity and Twall.

Table B: Identification of Model Types for the Corridor Data Set

Rank	R^2	Adj R^2	Std Error	F-statistic	Model
1	99.8	99.7	1.130	4140.87	Quad&Interacts (5V)
2	99.7	99.6	1.348	4751.89	Cubic, Interact. (3V)
3	97.6%	97.5%	3.54	895.68	Quad& Inter (3V)
4	96.77	----	4.1533	483.15	CART
5	99.4	99.3	1.851	1665.11	Cubic WO Indep (7V)
6	99.4	99.4	1.724	3843.80	Linear (6V)
7	99.1%	99.0%	2.209	2798.68	Linear 1 (5V)
8	85.2%	84.9%	8.72	257.21	Linear (3V)

Conclusions

A number of methods were considered in this analysis including classical multiple regression, polynomial regression, classification regression trees (CART), principal components, correlation matrix and residual analysis. Several graphical methods were used in model development and assessing model adequacy. In addition, several techniques were used to screen/identify underlying relationships. For example the matrix plot in Tables 3 and 3a suggested the inclusion of quadratic and interaction terms in our models. Whereas, principal components and best subset selection were used to screen and identify the main predictor variables. All of these methods helped to guide us in the selection of the predictor variables in our models. Using all of the above methods, several promising candidate models have been developed that may be used to predict the response variable, heat rate for the entry corridor data set. In the next phase of our research, validation of the adequacy of our models and other advanced methods will be explored.

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APPENDIX

Table 1: Corridor Data Set

No.	Altitude	Velocity	Twall	Pressure	Density	Temp	Mach	Dyn. Pres	Reynolds	Energy	Heat Rate
1	3.000E+02	2.500E+04	2.000E+03	2.973E-03	5.147E-09	3.364E+02	2.780E+01	1.608E+00	4.915E+02	8.042E+04	2.237E+01
2	3.000E+02	2.550E+04	2.000E+03	2.973E-03	5.147E-09	3.364E+02	2.836E+01	1.673E+00	5.013E+02	8.534E+04	2.384E+01
3	2.750E+02	2.450E+04	2.000E+03	1.144E-02	1.937E-08	3.441E+02	2.694E+01	5.814E+00	1.778E+03	2.849E+05	4.067E+01
4	2.750E+02	2.500E+04	2.000E+03	1.144E-02	1.937E-08	3.441E+02	2.749E+01	6.054E+00	1.814E+03	3.027E+05	4.340E+01
5	2.500E+02	2.100E+04	2.000E+03	4.111E-02	6.457E-08	3.709E+02	2.224E+01	1.424E+01	4.762E+03	5.979E+05	4.519E+01
6	2.500E+02	2.150E+04	2.000E+03	4.111E-02	6.457E-08	3.709E+02	2.277E+01	1.492E+01	4.876E+03	6.417E+05	4.876E+01
7	2.500E+02	2.200E+04	2.000E+03	4.111E-02	6.457E-08	3.709E+02	2.330E+01	1.563E+01	4.989E+03	6.875E+05	5.252E+01
8	2.500E+02	2.250E+04	2.000E+03	4.111E-02	6.457E-08	3.709E+02	2.383E+01	1.634E+01	5.102E+03	7.354E+05	5.646E+01
9	2.500E+02	2.300E+04	2.000E+03	4.111E-02	6.457E-08	3.709E+02	2.436E+01	1.708E+01	5.216E+03	7.856E+05	6.061E+01
10	2.500E+02	2.350E+04	2.000E+03	4.111E-02	6.457E-08	3.709E+02	2.489E+01	1.783E+01	5.329E+03	8.379E+05	6.495E+01
11	2.500E+02	2.400E+04	2.000E+03	4.111E-02	6.457E-08	3.709E+02	2.542E+01	1.860E+01	5.442E+03	8.926E+05	6.950E+01
12	2.250E+02	1.750E+04	2.000E+03	1.351E-01	1.956E-07	4.023E+02	1.780E+01	2.995E+01	1.123E+04	1.048E+06	4.348E+01
13	2.250E+02	1.800E+04	2.000E+03	1.351E-01	1.956E-07	4.023E+02	1.831E+01	3.169E+01	1.155E+04	1.141E+06	4.767E+01
14	2.250E+02	1.850E+04	2.000E+03	1.351E-01	1.956E-07	4.023E+02	1.881E+01	3.348E+01	1.187E+04	1.239E+06	5.213E+01
15	2.250E+02	1.900E+04	2.000E+03	1.351E-01	1.956E-07	4.023E+02	1.932E+01	3.531E+01	1.219E+04	1.342E+06	5.686E+01
16	2.250E+02	1.950E+04	2.000E+03	1.351E-01	1.956E-07	4.023E+02	1.983E+01	3.719E+01	1.252E+04	1.451E+06	6.187E+01
17	2.250E+02	2.000E+04	2.000E+03	1.351E-01	1.956E-07	4.023E+02	2.034E+01	3.912E+01	1.284E+04	1.565E+06	6.717E+01
18	2.250E+02	2.050E+04	2.000E+03	1.351E-01	1.956E-07	4.023E+02	2.085E+01	4.110E+01	1.316E+04	1.685E+06	7.276E+01
19	2.000E+02	1.300E+04	2.000E+03	4.023E-01	5.327E-07	4.399E+02	1.264E+01	4.501E+01	2.111E+04	1.170E+06	2.673E+01
20	2.000E+02	1.350E+04	2.000E+03	4.023E-01	5.327E-07	4.399E+02	1.313E+01	4.854E+01	2.192E+04	1.311E+06	3.036E+01
21	2.000E+02	1.400E+04	2.000E+03	4.023E-01	5.327E-07	4.399E+02	1.362E+01	5.220E+01	2.274E+04	1.462E+06	3.430E+01
22	2.000E+02	1.450E+04	2.000E+03	4.023E-01	5.327E-07	4.399E+02	1.410E+01	5.600E+01	2.355E+04	1.624E+06	3.857E+01
23	2.000E+02	1.500E+04	2.000E+03	4.023E-01	5.327E-07	4.399E+02	1.459E+01	5.993E+01	2.436E+04	1.798E+06	4.318E+01
24	2.000E+02	1.550E+04	2.000E+03	4.023E-01	5.327E-07	4.399E+02	1.507E+01	6.399E+01	2.517E+04	1.984E+06	4.814E+01
25	2.000E+02	1.600E+04	2.000E+03	4.023E-01	5.327E-07	4.399E+02	1.556E+01	6.819E+01	2.598E+04	2.182E+06	5.346E+01
26	2.000E+02	1.650E+04	2.000E+03	4.023E-01	5.327E-07	4.399E+02	1.605E+01	7.251E+01	2.680E+04	2.393E+06	5.916E+01
27	2.000E+02	1.700E+04	2.000E+03	4.023E-01	5.327E-07	4.399E+02	1.653E+01	7.698E+01	2.761E+04	2.617E+06	6.526E+01
28	2.000E+02	1.750E+04	2.000E+03	4.023E-01	5.327E-07	4.399E+02	1.702E+01	8.157E+01	2.842E+04	2.855E+06	7.176E+01
29	1.750E+02	9.500E+03	2.000E+03	1.098E+00	1.339E-06	4.776E+02	8.867E+00	6.042E+01	3.630E+04	1.148E+06	1.416E+01
30	1.750E+02	1.000E+04	2.000E+03	1.098E+00	1.339E-06	4.776E+02	9.333E+00	6.694E+01	3.821E+04	1.339E+06	1.705E+01
31	1.750E+02	1.050E+04	2.000E+03	1.098E+00	1.339E-06	4.776E+02	9.800E+00	7.380E+01	4.012E+04	1.550E+06	2.028E+01
32	1.750E+02	1.100E+04	2.000E+03	1.098E+00	1.339E-06	4.776E+02	1.027E+01	8.100E+01	4.203E+04	1.782E+06	2.388E+01
33	1.750E+02	1.150E+04	2.000E+03	1.098E+00	1.339E-06	4.776E+02	1.073E+01	8.853E+01	4.394E+04	2.036E+06	2.787E+01
34	1.750E+02	1.200E+04	2.000E+03	1.098E+00	1.339E-06	4.776E+02	1.120E+01	9.640E+01	4.585E+04	2.314E+06	3.227E+01
35	1.500E+02	6.500E+03	2.000E+03	2.842E+00	3.455E-06	4.791E+02	6.057E+00	7.299E+01	6.394E+04	9.489E+05	4.945E+00
36	1.500E+02	7.000E+03	2.000E+03	2.842E+00	3.455E-06	4.791E+02	6.523E+00	8.465E+01	6.886E+04	1.185E+06	6.884E+00

Table 1: Corridor Data Set

37	1.500E+02	7.500E+03	2.000E+03	2.842E+00	3.455E-06	4.791E+02	6.989E+00	9.718E+01	7.378E+04	1.458E+06	9.193E+00
38	1.500E+02	8.000E+03	2.000E+03	2.842E+00	3.455E-06	4.791E+02	7.455E+00	1.106E+02	7.870E+04	1.769E+06	1.190E+01
38	1.500E+02	8.500E+03	2.000E+03	2.842E+00	3.455E-06	4.791E+02	7.921E+00	1.248E+02	8.361E+04	2.122E+06	1.505E+01
40	1.500E+02	9.000E+03	2.000E+03	2.842E+00	3.455E-06	4.791E+02	8.387E+00	1.399E+02	8.853E+04	2.519E+06	1.865E+01
41	1.250E+02	5.000E+03	2.000E+03	7.769E+00	1.026E-05	4.412E+02	4.856E+00	1.282E+02	1.560E+05	1.282E+06	1.699E+00
42	1.250E+02	5.500E+03	2.000E+03	7.769E+00	1.026E-05	4.412E+02	5.341E+00	1.551E+02	1.716E+05	1.707E+06	3.455E+00
43	1.250E+02	6.000E+03	2.000E+03	7.769E+00	1.026E-05	4.412E+02	5.827E+00	1.846E+02	1.872E+05	2.216E+06	5.687E+00
44	1.000E+02	2.500E+03	2.000E+03	2.327E+01	3.318E-05	4.086E+02	2.523E+00	1.037E+02	2.686E+05	5.184E+05	6.013E-01
45	1.000E+02	3.000E+03	2.000E+03	2.327E+01	3.318E-05	4.086E+02	3.027E+00	1.493E+02	3.224E+05	8.958E+05	1.533E+00
46	1.000E+02	3.500E+03	2.000E+03	2.327E+01	3.318E-05	4.086E+02	3.532E+00	2.032E+02	3.761E+05	1.422E+06	2.883E+00
47	3.000E+02	2.500E+04	2.500E+03	2.973E-03	5.147E-09	3.364E+02	2.780E+01	1.608E+00	4.915E+02	8.042E+04	2.215E+01
48	3.000E+02	2.550E+04	2.500E+03	2.973E-03	5.147E-09	3.364E+02	2.836E+01	1.673E+00	5.013E+02	8.534E+04	2.361E+01
49	2.750E+02	2.450E+04	2.500E+03	1.144E-02	1.937E-08	3.441E+02	2.694E+01	5.814E+00	1.778E+03	2.849E+05	4.025E+01
50	2.750E+02	2.500E+04	2.500E+03	1.144E-02	1.937E-08	3.441E+02	2.749E+01	6.054E+00	1.814E+03	3.027E+05	4.297E+01
51	2.500E+02	2.100E+04	2.500E+03	4.111E-02	6.457E-08	3.709E+02	2.224E+01	1.424E+01	4.762E+03	5.979E+05	4.455E+01
52	2.500E+02	2.150E+04	2.500E+03	4.111E-02	6.457E-08	3.709E+02	2.277E+01	1.492E+01	4.876E+03	6.417E+05	4.810E+01
53	2.500E+02	2.200E+04	2.500E+03	4.111E-02	6.457E-08	3.709E+02	2.330E+01	1.563E+01	4.989E+03	6.875E+05	5.184E+01
54	2.500E+02	2.250E+04	2.500E+03	4.111E-02	6.457E-08	3.709E+02	2.383E+01	1.634E+01	5.102E+03	7.354E+05	5.577E+01
55	2.500E+02	2.300E+04	2.500E+03	4.111E-02	6.457E-08	3.709E+02	2.436E+01	1.708E+01	5.216E+03	7.856E+05	5.989E+01
56	2.500E+02	2.350E+04	2.500E+03	4.111E-02	6.457E-08	3.709E+02	2.489E+01	1.783E+01	5.329E+03	8.379E+05	6.422E+01
57	2.500E+02	2.400E+04	2.500E+03	4.111E-02	6.457E-08	3.709E+02	2.542E+01	1.860E+01	5.442E+03	8.926E+05	6.875E+01
58	2.250E+02	1.750E+04	2.500E+03	1.351E-01	1.956E-07	4.023E+02	1.780E+01	2.995E+01	1.123E+04	1.048E+06	4.257E+01
59	2.250E+02	1.800E+04	2.500E+03	1.351E-01	1.956E-07	4.023E+02	1.831E+01	3.169E+01	1.155E+04	1.141E+06	4.674E+01
60	2.250E+02	1.850E+04	2.500E+03	1.351E-01	1.956E-07	4.023E+02	1.881E+01	3.348E+01	1.187E+04	1.239E+06	5.116E+01
61	2.250E+02	1.900E+04	2.500E+03	1.351E-01	1.956E-07	4.023E+02	1.932E+01	3.531E+01	1.219E+04	1.342E+06	5.586E+01
62	2.250E+02	1.950E+04	2.500E+03	1.351E-01	1.956E-07	4.023E+02	1.983E+01	3.719E+01	1.252E+04	1.451E+06	6.084E+01
63	2.250E+02	2.000E+04	2.500E+03	1.351E-01	1.956E-07	4.023E+02	2.034E+01	3.912E+01	1.284E+04	1.565E+06	6.611E+01
64	2.250E+02	2.050E+04	2.500E+03	1.351E-01	1.956E-07	4.023E+02	2.085E+01	4.110E+01	1.316E+04	1.685E+06	7.167E+01
65	2.000E+02	1.300E+04	2.500E+03	4.023E-01	5.327E-07	4.399E+02	1.264E+01	4.501E+01	2.111E+04	1.170E+06	2.566E+01
66	2.000E+02	1.350E+04	2.500E+03	4.023E-01	5.327E-07	4.399E+02	1.313E+01	4.854E+01	2.192E+04	1.311E+06	2.924E+01
67	2.000E+02	1.400E+04	2.500E+03	4.023E-01	5.327E-07	4.399E+02	1.362E+01	5.220E+01	2.274E+04	1.462E+06	3.314E+01
68	2.000E+02	1.450E+04	2.500E+03	4.023E-01	5.327E-07	4.399E+02	1.410E+01	5.600E+01	2.355E+04	1.624E+06	3.736E+01
69	2.000E+02	1.500E+04	2.500E+03	4.023E-01	5.327E-07	4.399E+02	1.459E+01	5.993E+01	2.436E+04	1.798E+06	4.192E+01
70	2.000E+02	1.550E+04	2.500E+03	4.023E-01	5.327E-07	4.399E+02	1.507E+01	6.399E+01	2.517E+04	1.984E+06	4.683E+01
71	2.000E+02	1.600E+04	2.500E+03	4.023E-01	5.327E-07	4.399E+02	1.556E+01	6.819E+01	2.598E+04	2.182E+06	5.210E+01
72	2.000E+02	1.650E+04	2.500E+03	4.023E-01	5.327E-07	4.399E+02	1.605E+01	7.251E+01	2.680E+04	2.393E+06	5.776E+01
73	2.000E+02	1.700E+04	2.500E+03	4.023E-01	5.327E-07	4.399E+02	1.653E+01	7.698E+01	2.761E+04	2.617E+06	6.380E+01
74	2.000E+02	1.750E+04	2.500E+03	4.023E-01	5.327E-07	4.399E+02	1.702E+01	8.157E+01	2.842E+04	2.855E+06	7.026E+01

Table 1: Corridor Data Set

75	1.750E+02	9.500E+03	2.500E+03	1.098E+00	1.339E-06	4.776E+02	8.867E+00	6.042E+01	3.630E+04	1.148E+06	1.298E+01
76	1.750E+02	1.000E+04	2.500E+03	1.098E+00	1.339E-06	4.776E+02	9.333E+00	6.694E+01	3.821E+04	1.339E+06	1.579E+01
77	1.750E+02	1.050E+04	2.500E+03	1.098E+00	1.339E-06	4.776E+02	9.800E+00	7.380E+01	4.012E+04	1.550E+06	1.895E+01
78	1.750E+02	1.100E+04	2.500E+03	1.098E+00	1.339E-06	4.776E+02	1.027E+01	8.100E+01	4.203E+04	1.782E+06	2.248E+01
79	1.750E+02	1.150E+04	2.500E+03	1.098E+00	1.339E-06	4.776E+02	1.073E+01	8.853E+01	4.394E+04	2.036E+06	2.640E+01
80	1.750E+02	1.200E+04	2.500E+03	1.098E+00	1.339E-06	4.776E+02	1.120E+01	9.640E+01	4.585E+04	2.314E+06	3.073E+01
81	1.500E+02	6.500E+03	2.500E+03	2.842E+00	3.455E-06	4.791E+02	6.057E+00	7.299E+01	6.394E+04	9.489E+05	3.705E+00
82	1.500E+02	7.000E+03	2.500E+03	2.842E+00	3.455E-06	4.791E+02	6.523E+00	8.465E+01	6.886E+04	1.185E+06	5.538E+00
83	1.500E+02	7.500E+03	2.500E+03	2.842E+00	3.455E-06	4.791E+02	6.989E+00	9.718E+01	7.378E+04	1.458E+06	7.739E+00
84	1.500E+02	8.000E+03	2.500E+03	2.842E+00	3.455E-06	4.791E+02	7.455E+00	1.106E+02	7.870E+04	1.769E+06	1.034E+01
85	1.500E+02	8.500E+03	2.500E+03	2.842E+00	3.455E-06	4.791E+02	7.921E+00	1.248E+02	8.361E+04	2.122E+06	1.337E+01
86	1.500E+02	9.000E+03	2.500E+03	2.842E+00	3.455E-06	4.791E+02	8.387E+00	1.399E+02	8.853E+04	2.519E+06	1.687E+01
87	1.250E+02	5.000E+03	2.500E+03	7.769E+00	1.026E-05	4.412E+02	4.856E+00	1.282E+02	1.560E+05	1.282E+06	6.960E-02
88	1.250E+02	5.500E+03	2.500E+03	7.769E+00	1.026E-05	4.412E+02	5.341E+00	1.551E+02	1.716E+05	1.707E+06	1.652E+00
89	1.250E+02	6.000E+03	2.500E+03	7.769E+00	1.026E-05	4.412E+02	5.827E+00	1.846E+02	1.872E+05	2.216E+06	3.708E+00
90	1.000E+02	2.500E+03	2.500E+03	2.327E+01	3.318E-05	4.086E+02	2.523E+00	1.037E+02	2.686E+05	5.184E+05	2.337E+00
91	1.000E+02	3.000E+03	2.500E+03	2.327E+01	3.318E-05	4.086E+02	3.027E+00	1.493E+02	3.224E+05	8.958E+05	8.276E-01
92	1.000E+02	3.500E+03	2.500E+03	2.327E+01	3.318E-05	4.086E+02	3.532E+00	2.032E+02	3.761E+05	1.422E+06	2.085E+00
93	3.000E+02	2.500E+04	3.000E+03	2.973E-03	5.147E-09	3.364E+02	2.780E+01	1.608E+00	4.915E+02	8.042E+04	2.193E+01
94	3.000E+02	2.550E+04	3.000E+03	2.973E-03	5.147E-09	3.364E+02	2.836E+01	1.673E+00	5.013E+02	8.534E+04	2.338E+01
95	2.750E+02	2.450E+04	3.000E+03	1.144E-02	1.937E-08	3.441E+02	2.694E+01	5.814E+00	1.778E+03	2.849E+05	3.983E+01
96	2.750E+02	2.500E+04	3.000E+03	1.144E-02	1.937E-08	3.441E+02	2.749E+01	6.054E+00	1.814E+03	3.027E+05	4.254E+01
97	2.500E+02	2.100E+04	3.000E+03	4.111E-02	6.457E-08	3.709E+02	2.224E+01	1.424E+01	4.762E+03	5.979E+05	4.390E+01
98	2.500E+02	2.150E+04	3.000E+03	4.111E-02	6.457E-08	3.709E+02	2.277E+01	1.492E+01	4.876E+03	6.417E+05	4.744E+01
99	2.500E+02	2.200E+04	3.000E+03	4.111E-02	6.457E-08	3.709E+02	2.330E+01	1.563E+01	4.989E+03	6.875E+05	5.116E+01
100	2.500E+02	2.250E+04	3.000E+03	4.111E-02	6.457E-08	3.709E+02	2.383E+01	1.634E+01	5.102E+03	7.354E+05	5.507E+01
101	2.500E+02	2.300E+04	3.000E+03	4.111E-02	6.457E-08	3.709E+02	2.436E+01	1.708E+01	5.216E+03	7.856E+05	5.918E+01
102	2.500E+02	2.350E+04	3.000E+03	4.111E-02	6.457E-08	3.709E+02	2.489E+01	1.783E+01	5.329E+03	8.379E+05	6.348E+01
103	2.500E+02	2.400E+04	3.000E+03	4.111E-02	6.457E-08	3.709E+02	2.542E+01	1.860E+01	5.442E+03	8.926E+05	6.800E+01
104	2.250E+02	1.750E+04	3.000E+03	1.351E-01	1.956E-07	4.023E+02	1.780E+01	2.995E+01	1.123E+04	1.048E+06	4.166E+01
105	2.250E+02	1.800E+04	3.000E+03	1.351E-01	1.956E-07	4.023E+02	1.831E+01	3.169E+01	1.155E+04	1.141E+06	4.580E+01
106	2.250E+02	1.850E+04	3.000E+03	1.351E-01	1.956E-07	4.023E+02	1.881E+01	3.348E+01	1.187E+04	1.239E+06	5.019E+01
107	2.250E+02	1.900E+04	3.000E+03	1.351E-01	1.956E-07	4.023E+02	1.932E+01	3.531E+01	1.219E+04	1.342E+06	5.486E+01
108	2.250E+02	1.950E+04	3.000E+03	1.351E-01	1.956E-07	4.023E+02	1.983E+01	3.719E+01	1.252E+04	1.451E+06	5.981E+01
109	2.250E+02	2.000E+04	3.000E+03	1.351E-01	1.956E-07	4.023E+02	2.034E+01	3.912E+01	1.284E+04	1.565E+06	6.505E+01
110	2.250E+02	2.050E+04	3.000E+03	1.351E-01	1.956E-07	4.023E+02	2.085E+01	4.110E+01	1.316E+04	1.685E+06	7.058E+01
111	2.000E+02	1.300E+04	3.000E+03	4.023E-01	5.327E-07	4.399E+02	1.264E+01	4.501E+01	2.111E+04	1.170E+06	2.459E+01
112	2.000E+02	1.350E+04	3.000E+03	4.023E-01	5.327E-07	4.399E+02	1.313E+01	4.854E+01	2.192E+04	1.311E+06	2.813E+01

Table 1: Corridor Data Set

113	2.000E+02	1.400E+04	3.000E+03	4.023E-01	5.327E-07	4.399E+02	1.362E+01	5.220E+01	2.274E+04	1.462E+06	3.198E+01
114	2.000E+02	1.450E+04	3.000E+03	4.023E-01	5.327E-07	4.399E+02	1.410E+01	5.600E+01	2.355E+04	1.624E+06	3.615E+01
115	2.000E+02	1.500E+04	3.000E+03	4.023E-01	5.327E-07	4.399E+02	1.459E+01	5.993E+01	2.436E+04	1.798E+06	4.066E+01
116	2.000E+02	1.550E+04	3.000E+03	4.023E-01	5.327E-07	4.399E+02	1.507E+01	6.399E+01	2.517E+04	1.984E+06	4.552E+01
117	2.000E+02	1.600E+04	3.000E+03	4.023E-01	5.327E-07	4.399E+02	1.556E+01	6.819E+01	2.598E+04	2.182E+06	5.075E+01
118	2.000E+02	1.650E+04	3.000E+03	4.023E-01	5.327E-07	4.399E+02	1.605E+01	7.251E+01	2.680E+04	2.393E+06	5.636E+01
119	2.000E+02	1.700E+04	3.000E+03	4.023E-01	5.327E-07	4.399E+02	1.653E+01	7.698E+01	2.761E+04	2.617E+06	6.235E+01
120	2.000E+02	1.750E+04	3.000E+03	4.023E-01	5.327E-07	4.399E+02	1.702E+01	8.157E+01	2.842E+04	2.855E+06	6.876E+01
121	1.750E+02	9.500E+03	3.000E+03	1.098E+00	1.339E-06	4.776E+02	8.867E+00	6.042E+01	3.630E+04	1.148E+06	1.180E+01
122	1.750E+02	1.000E+04	3.000E+03	1.098E+00	1.339E-06	4.776E+02	9.333E+00	6.694E+01	3.821E+04	1.339E+06	1.454E+01
123	1.750E+02	1.050E+04	3.000E+03	1.098E+00	1.339E-06	4.776E+02	9.800E+00	7.380E+01	4.012E+04	1.550E+06	1.763E+01
124	1.750E+02	1.100E+04	3.000E+03	1.098E+00	1.339E-06	4.776E+02	1.027E+01	8.100E+01	4.203E+04	1.782E+06	2.109E+01
125	1.750E+02	1.150E+04	3.000E+03	1.098E+00	1.339E-06	4.776E+02	1.073E+01	8.853E+01	4.394E+04	2.036E+06	2.493E+01
126	1.750E+02	1.200E+04	3.000E+03	1.098E+00	1.339E-06	4.776E+02	1.120E+01	9.640E+01	4.585E+04	2.314E+06	2.919E+01
127	1.500E+02	6.500E+03	3.000E+03	2.842E+00	3.455E-06	4.791E+02	6.057E+00	7.299E+01	6.394E+04	9.489E+05	2.466E+00
128	1.500E+02	7.000E+03	3.000E+03	2.842E+00	3.455E-06	4.791E+02	6.523E+00	8.465E+01	6.886E+04	1.185E+06	4.191E+00
129	1.500E+02	7.500E+03	3.000E+03	2.842E+00	3.455E-06	4.791E+02	6.989E+00	9.718E+01	7.378E+04	1.458E+06	6.284E+00
130	1.500E+02	8.000E+03	3.000E+03	2.842E+00	3.455E-06	4.791E+02	7.455E+00	1.106E+02	7.870E+04	1.769E+06	8.775E+00
131	1.500E+02	8.500E+03	3.000E+03	2.842E+00	3.455E-06	4.791E+02	7.921E+00	1.248E+02	8.361E+04	2.122E+06	1.170E+01
132	1.500E+02	9.000E+03	3.000E+03	2.842E+00	3.455E-06	4.791E+02	8.387E+00	1.399E+02	8.853E+04	2.519E+06	1.508E+01
133	1.250E+02	5.000E+03	3.000E+03	7.769E+00	1.026E-05	4.412E+02	4.856E+00	1.282E+02	1.560E+05	1.282E+06	8.843E-01
134	1.250E+02	5.500E+03	3.000E+03	7.769E+00	1.026E-05	4.412E+02	5.341E+00	1.551E+02	1.716E+05	1.707E+06	2.553E+00
135	1.250E+02	6.000E+03	3.000E+03	7.769E+00	1.026E-05	4.412E+02	5.827E+00	1.846E+02	1.872E+05	2.216E+06	1.729E+00
136	1.000E+02	2.500E+03	3.000E+03	2.327E+01	3.318E-05	4.086E+02	2.523E+00	1.037E+02	2.686E+05	5.184E+05	2.179E+00
137	1.000E+02	3.000E+03	3.000E+03	2.327E+01	3.318E-05	4.086E+02	3.027E+00	1.493E+02	3.224E+05	8.958E+05	1.221E-01
138	1.000E+02	3.500E+03	3.000E+03	2.327E+01	3.318E-05	4.086E+02	3.532E+00	2.032E+02	3.761E+05	1.422E+06	1.288E+00

Table 2: Summary Statistics for Corridor Data

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Altitude	138	197.83	200.00	197.78	50.95	4.34
Velocity	138	14772	15250	14855	6741	574
Twall	138	2500.0	2500.0	2500.0	409.7	34.9
Heat Rate	138	34.17	35.23	33.99	22.43	1.91

Variable	Minimum	Maximum	Q1	Q3
Altitude	100.00	300.00	150.00	225.00
Velocity	2500	25500	9000	20500
Twall	2000.0	3000.0	2000.0	3000.0
Heat Rate	0.07	72.76	13.96	52.76

Table 2a: Summary Statistics Full Data

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Altitude	1269	200.00	200.00	200.00	64.58	1.81
Velocity	1269	14000	14000	14000	6785	190
Twall	1269	2500.0	2500.0	2500.0	408.4	11.5
Heat Rate	1269	131.13	22.37	83.46	273.06	7.67

Variable	Minimum	Maximum	Q1	Q3
Altitude	100.00	300.00	150.00	250.00
Velocity	2500	25500	8000	20000
Twall	2000.0	3000.0	2000.0	3000.0
Heat Rate	0.00	1914.00	2.87	117.80

Table 3: Matrix Plot for Corridor Data

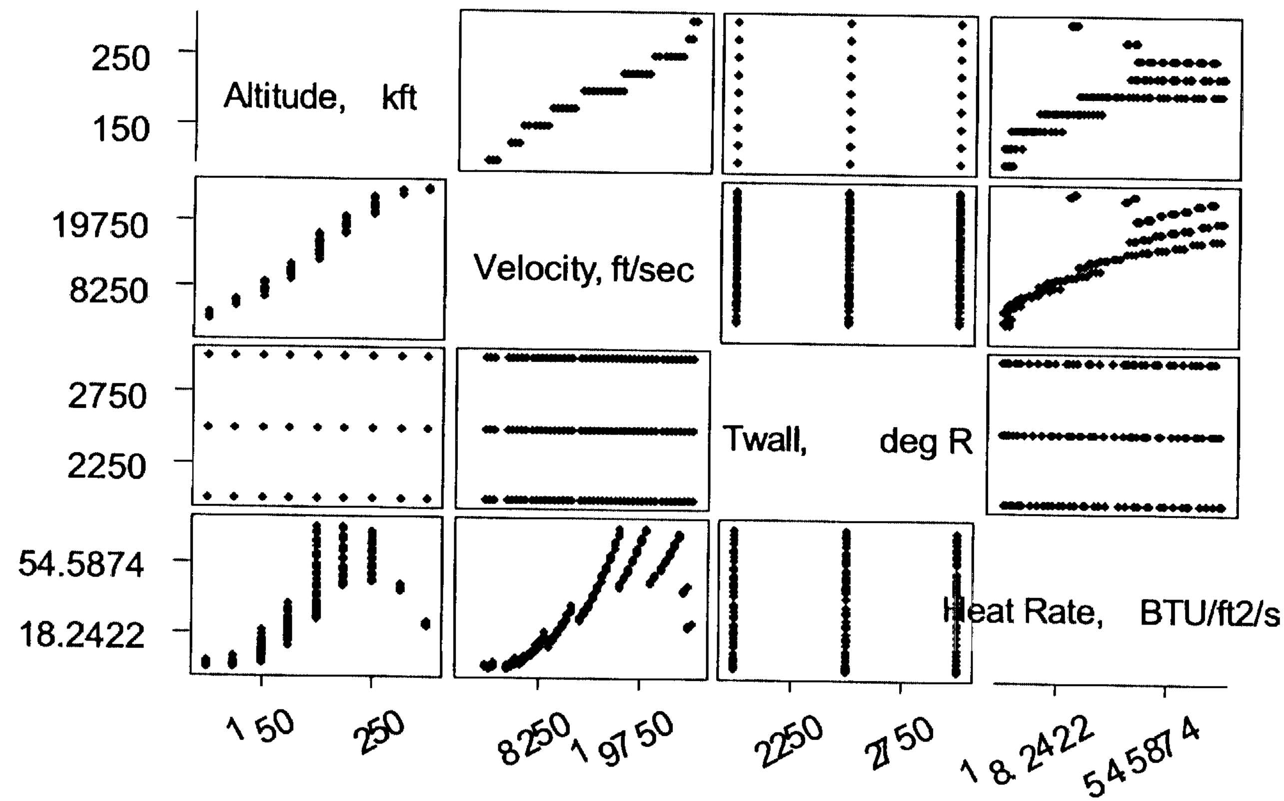

Table 3a: Matrix Plot for Full Data

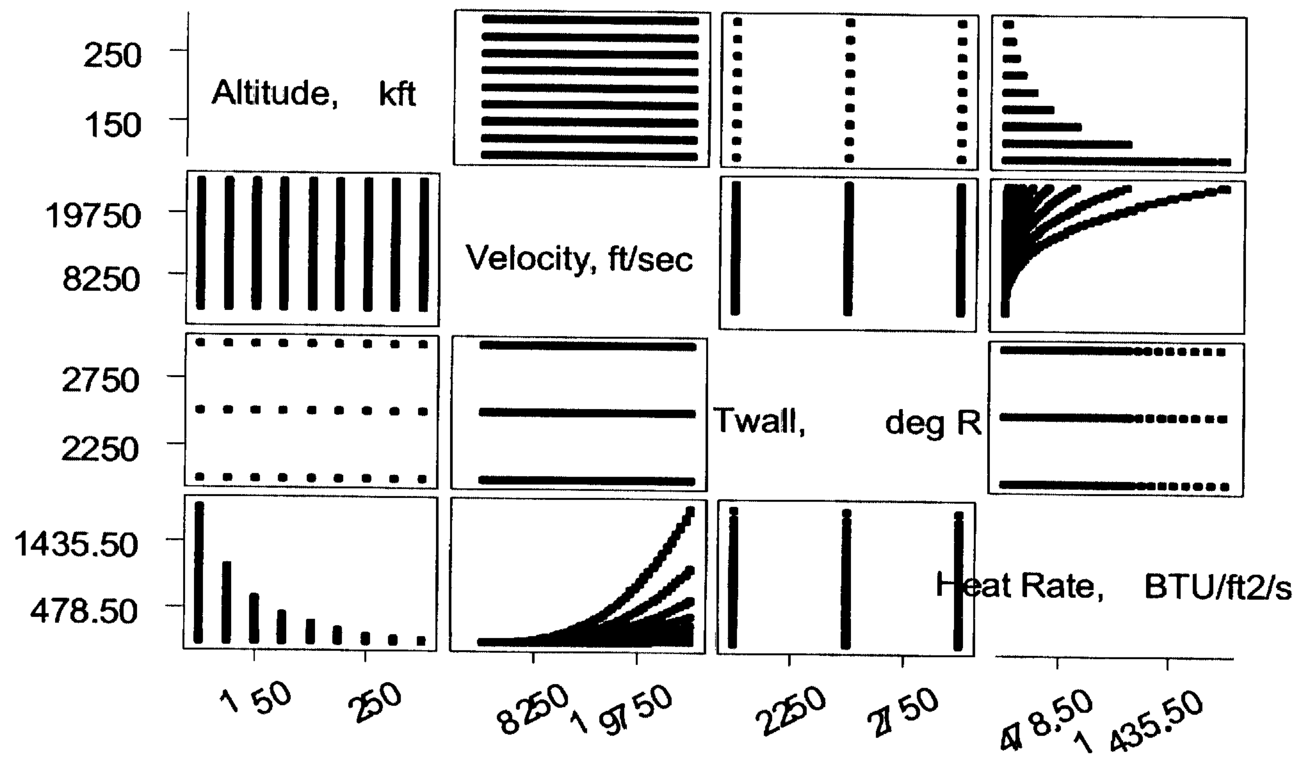

Table 4: Correlation Matrix for Corridor Data

	Altitude	Velocity	Density	Temp	Dyn.Pres	Reynolds	Energy
Velocity	0.980						
	0.000						
Density	-0.682	-0.639					
	0.000	0.000					
Temp	-0.731	-0.759	0.045				
	0.000	0.000	0.603				
Dyn.Pres	-0.921	-0.879	0.666	0.614			
	0.000	0.000	0.000	0.000			
Reynolds	-0.797	-0.756	0.962	0.204	0.829		
	0.000	0.000	0.000	0.017	0.000		
Energy	-0.471	-0.388	-0.077	0.707	0.552	0.085	
	0.000	0.000	0.367	0.000	0.000	0.321	
Heat Rate	0.703	0.809	-0.561	-0.467	-0.658	-0.656	0.065
	0.000	0.000	0.000	0.000	0.000	0.000	0.448

Cell Contents: Pearson correlation

P-Value

Table 4a: Correlation Matrix for Full Data

	Altitude	Velocity	Density,	Temp,	Mach	Dyn. Pre	Reynolds	Energy,
Velocity	0.000							
	1.000							
Density,	-0.717	0.000						
	0.000	1.000						
Temp,	-0.749	-0.000	0.120					
	0.000	1.000	0.000					
Mach	0.099	0.990	-0.020	-0.127				
	0.000	0.000	0.485	0.000				
Dyn. Pre	-0.530	0.308	0.740	0.089	0.284			
	0.000	0.000	0.000	0.001	0.000			
Reynolds	-0.619	0.220	0.878	0.095	0.198	0.964		
	0.000	0.000	0.000	0.001	0.000	0.000		
Energy,	-0.463	0.333	0.647	0.078	0.311	0.985	0.909	
	0.000	0.000	0.000	0.006	0.000	0.000	0.000	
Heat Rat	-0.490	0.493	0.543	0.195	0.447	0.919	0.817	0.949
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Cell Contents: Pearson correlation

P-Value

Table 5a: Principal Components Analysis

Importance of components:

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5
Standard deviation	2.5062969	1.4533400	1.0000000	0.69195297	0.312497616
Proportion of Variance	0.6281524	0.2112197	0.1000000	0.04787989	0.009765476
Cumulative Proportion	0.6281524	0.8393721	0.9393721	0.98725200	0.997017475

	Comp.6	Comp.7	Comp.8	Comp.9
Standard deviation	0.140146523	0.0896247148	0.045035427	0.01084377167
Proportion of Variance	0.001964105	0.0008032589	0.000202819	0.00001175874
Cumulative Proportion	0.998981580	0.9997848387	0.999987658	0.99999941638

	Comp.10
Standard deviation	2.415833e-003
Proportion of Variance	5.836249e-007
Cumulative Proportion	1.000000e+000

Table 5b: Principal Components Analysis

Loadings:

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8	Comp.9	Comp.10
Altitude	-0.392			-0.110		0.564	-0.709			
Velocity	-0.383			-0.335		0.180	0.440	-0.214	0.621	0.255
Twall			1.000							
Pressure	0.317	-0.408		-0.109	-0.335	0.121		0.328	-0.176	0.675
Density	0.311	-0.418		-0.120	-0.397	0.159		0.157	0.309	-0.639
Temp	0.253	0.503		0.317	-0.243	0.672	0.264			
Mach	-0.379	-0.148		-0.310		0.233	0.435	0.234	-0.609	-0.260
Dyn.Pres	0.377			-0.304	0.716	0.220		0.388	0.213	
Reynolds	0.352	-0.309		-0.176	0.209	0.180		-0.781	-0.249	
Energy	0.166	0.516		-0.728	-0.320	-0.154	-0.193			

Table 6: Best Subset Selection

Response is Heat Rate

Vars	R-Sq	R-Sq(adj)	C-p	S	Altitude	Velocity	Twall,	Density,	Temp,	Mach	Dyn. Pres	Reynolds,	Energy,
1	65.4	65.2	2E+04	13.233		X							
1	58.8	58.5	2E+04	14.446						X			
2	85.1	84.8	6624.2	8.7306	X	X							
2	84.8	84.6	6731.8	8.7998		X				X			
3	97.5	97.4	1006.9	3.5947	X	X				X			
3	93.7	93.6	2710.5	5.6821		X				X		X	
4	98.7	98.7	455.9	2.5858	X	X				X	X		
4	98.6	98.6	487.3	2.6546	X	X				X			X
5	99.3	99.3	192.0	1.9155	X	X			X	X	X		
5	99.1	99.0	296.7	2.2086	X	X				X	X	X	
6	99.4	99.4	131.5	1.7236	X	X	X		X	X	X		
6	99.4	99.3	167.0	1.8394	X	X		X			X	X	X
7	99.5	99.5	106.5	1.6362	X	X	X	X			X	X	X
7	99.5	99.5	113.7	1.6618	X	X		X		X	X	X	X
8	99.6	99.6	53.2	1.4300	X	X	X	X		X	X	X	X
8	99.6	99.6	70.5	1.4997	X	X		X	X	X	X	X	X
9	99.7	99.7	10.0	1.2341	X	X	X	X	X	X	X	X	X

Table 7a: CART MODEL

```
Regression tree:
tree(formula = Heat.Rate ~ Altitude + Velocity + Twall + Pressure + Density +
        Temp + Mach + Dyn.Pres + Reynolds + Energy, data = cpsphere, na.action
         = na.exclude, mincut = 5, minsize = 10, mindev = 0.01)
Variables actually used in tree construction:
[1] "Velocity" "Mach"     "Altitude" "Energy"   "Dyn.Pres"
Number of terminal nodes:  9
Residual mean deviance:  17.25 = 2225 / 129

Distribution of residuals:
    Min.  1st Qu.  Median    Mean  3rd Qu.    Max.
 -9.3730 -2.6180 -0.3792  0.0000  2.8720  8.8710
node), split, n, deviance, yval
       * denotes terminal node
```

Table 7b: CART MODEL

```
1) root 138 68900.000 34.170
  2) Velocity<13750 60  6000.000 12.580
    4) Mach<7.6881 30   293.000  3.909 *
    5) Mach>7.6881 30  1197.000 21.250
     10) Velocity<10750 15    95.580 15.590 *
     11) Velocity>10750 15   142.100 26.900 *
  3) Velocity>13750 78 13420.000 50.780
    6) Altitude<287.5 72  8354.000 53.100
     12) Velocity<15750 12   324.800 39.810 *
     13) Velocity>15750 60  5486.000 55.760
       26) Energy<711465 15   248.700 45.540 *
       27) Energy>711465 45  3149.000 59.170
         54) Mach<19.577 24  1717.000 55.030
          108) Dyn.Pres<74.7445 18   494.300 51.030 *
          109) Dyn.Pres>74.7445 6    71.170 67.030 *
         55) Mach>19.577 21   552.300 63.890 *
    7) Altitude>287.5 6     3.405 22.880 *
```

Figure 1. Classification and Regression Tree for Corridor Data

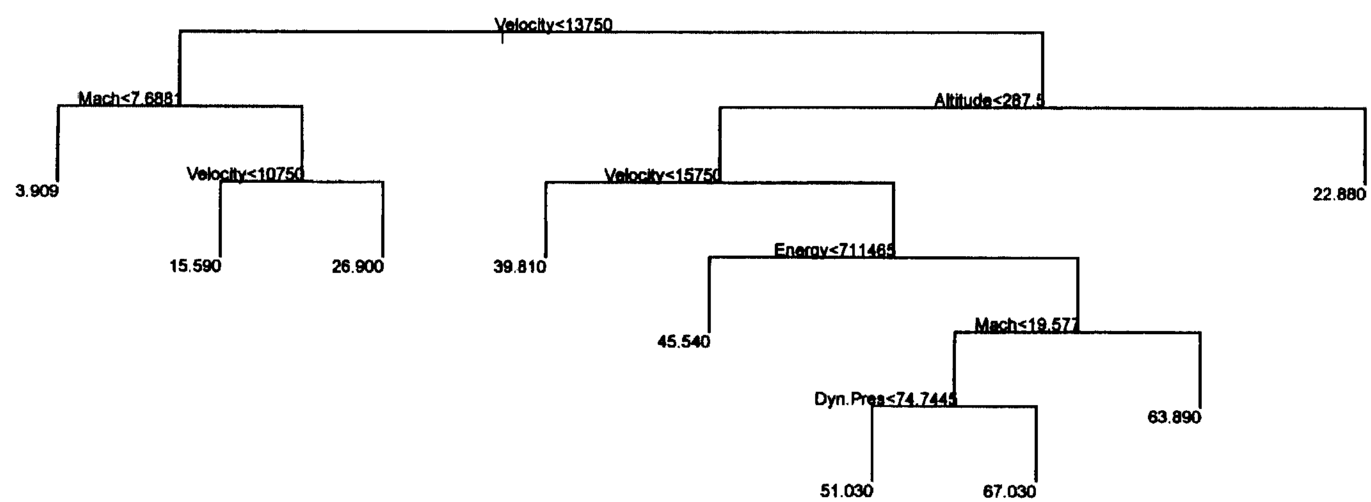

Table 8: Regression Analysis: Heat Rate (Corridor Data 3V)

The regression equation is

```
Heat Rate,    BTU/ft2/s = 89.6 - 2.12 Altitude,    kft
          + 0.0221 Velocity, ft/sec - 0.00203 Twall,       deg R
          + 0.0130 Alt*2 +0.000001 Alt*3 +0.000002 Vel*2 -0.000000 Vel*3
          -0.000297 alt*Vel +0.000000 Vel*Alt*2
```

Predictor	Coef	SE Coef	T	P
Constant	89.59	36.07	2.48	0.014
Altitude	-2.1228	0.9511	-2.23	0.027
Velocity	0.022133	0.006144	3.60	0.000
Twall	-0.0020343	0.0002812	-7.24	0.000
Alt*2	0.013035	0.007535	1.73	0.086
Alt*3	0.00000099	0.00001611	0.06	0.951
Vel*2	0.00000194	0.00000022	8.82	0.000
Vel*3	-0.00000000	0.00000000	-5.26	0.000
alt*Vel	-0.00029731	0.00008705	-3.42	0.001
Vel*Alt*	0.00000008	0.00000019	0.41	0.683

S = 1.348   R-Sq = 99.7%   R-Sq(adj) = 99.6%

Table 9: Analysis of Variance (3V)

Source	DF	SS	MS	F	P
Regression	8	68669.1	8583.6	4751.89	0.000
Residual Error	129	233.0	1.8		
Total	137	68902.1			

Table 10: Unusual Observations (3V)

Obs	Altitude	Heat Rat	Fit	SE Fit	Residual	St Resid
1	300	22.370	24.584	0.636	-2.214	-1.86 X
5	250	45.190	48.235	0.315	-3.045	-2.32R
51	250	44.550	47.218	0.282	-2.668	-2.02R
89	125	3.708	7.270	0.303	-3.562	-2.71R
90	100	2.337	-1.643	0.522	3.980	3.20R
93	300	21.930	22.550	0.636	-0.620	-0.52 X
94	300	23.380	20.829	0.531	2.551	2.06R
96	275	42.540	39.504	0.496	3.036	2.42R
135	125	1.729	6.253	0.334	-4.524	-3.46R
136	100	2.179	-2.660	0.540	4.839	3.92R

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Figure 2. Histogram of Residuals

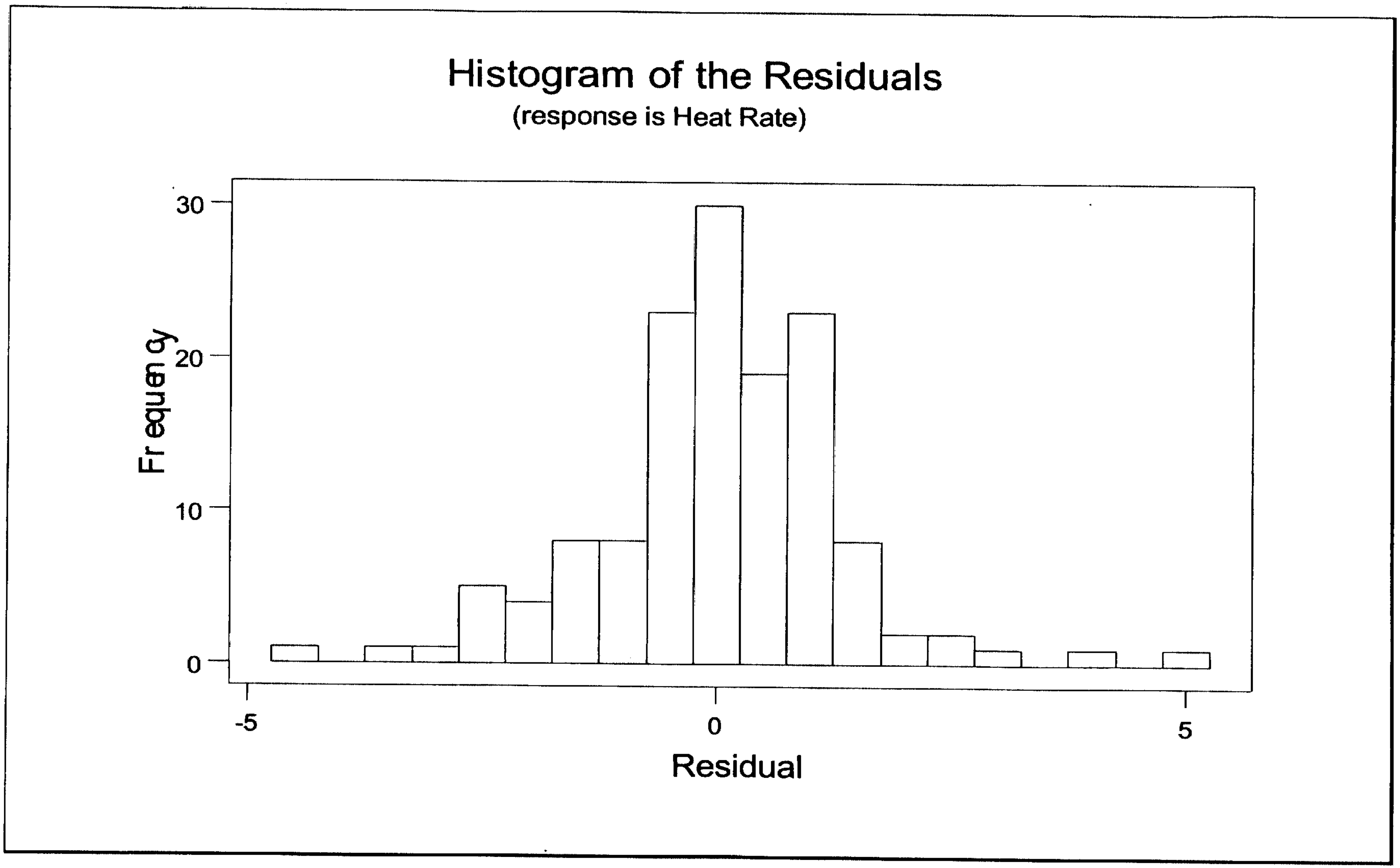

Figure 3. Normal Probability Plot (3V)

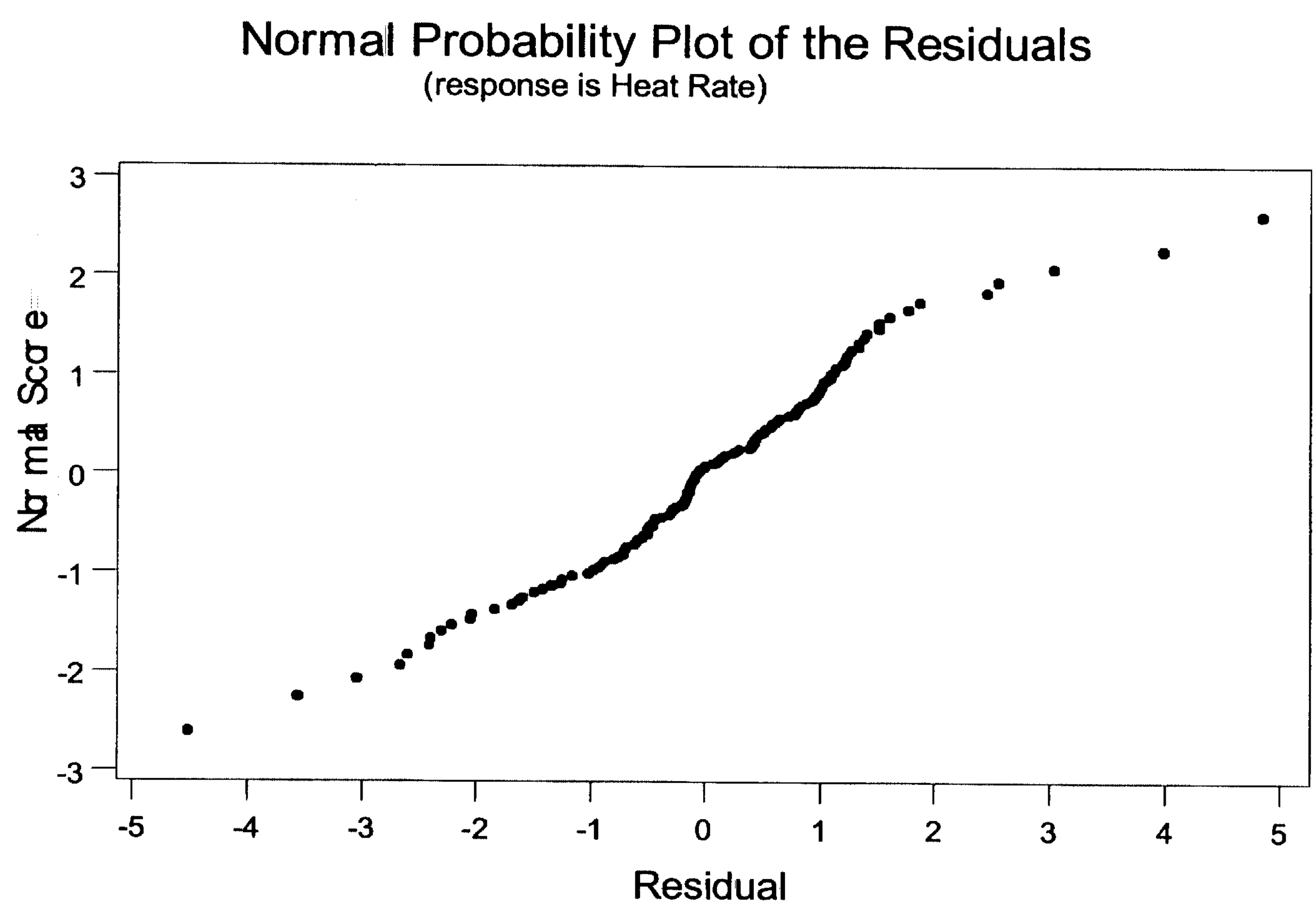

Figure 4. Residual Plot (3V)

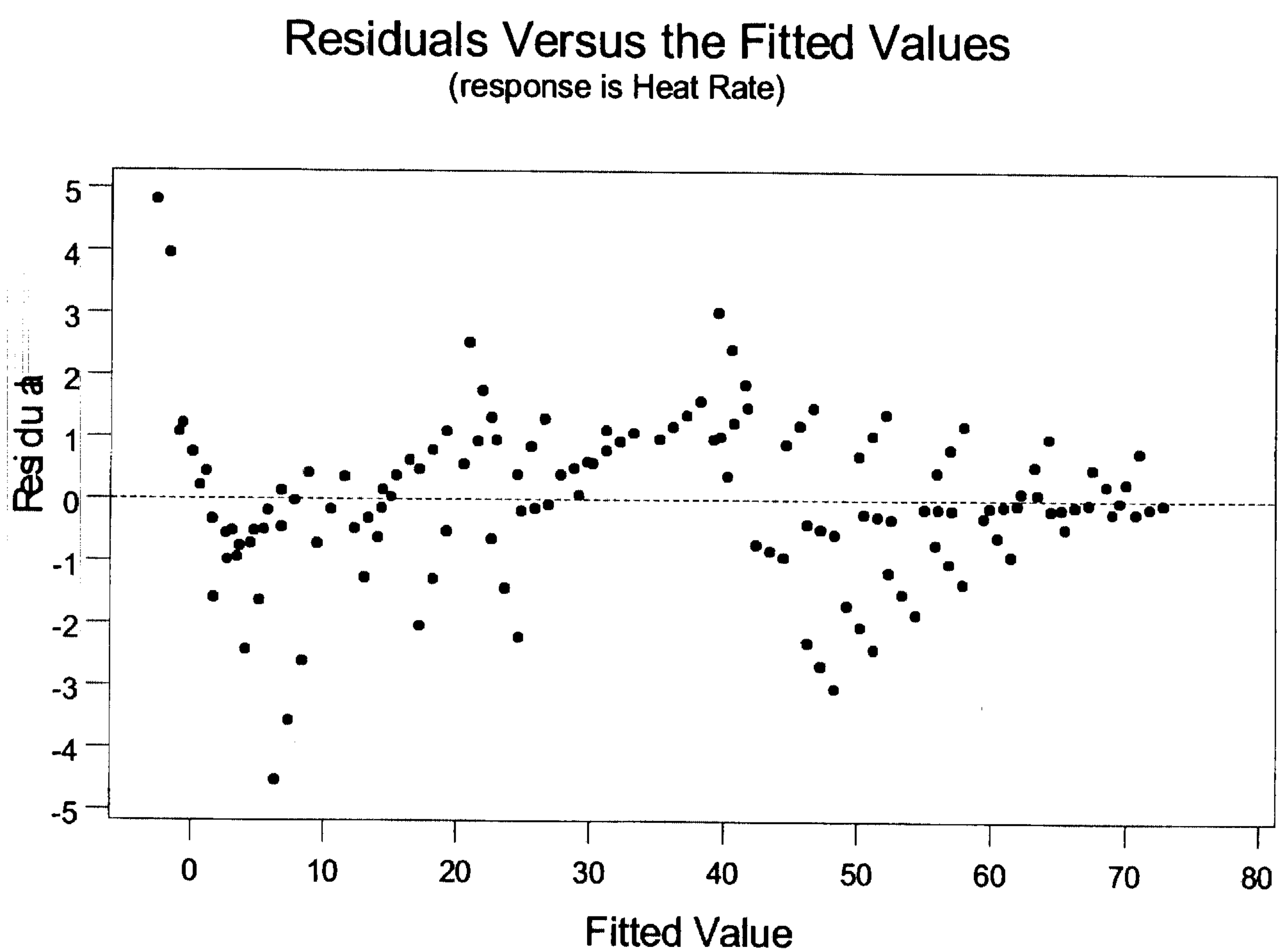

Figure 5. 3-D Plot

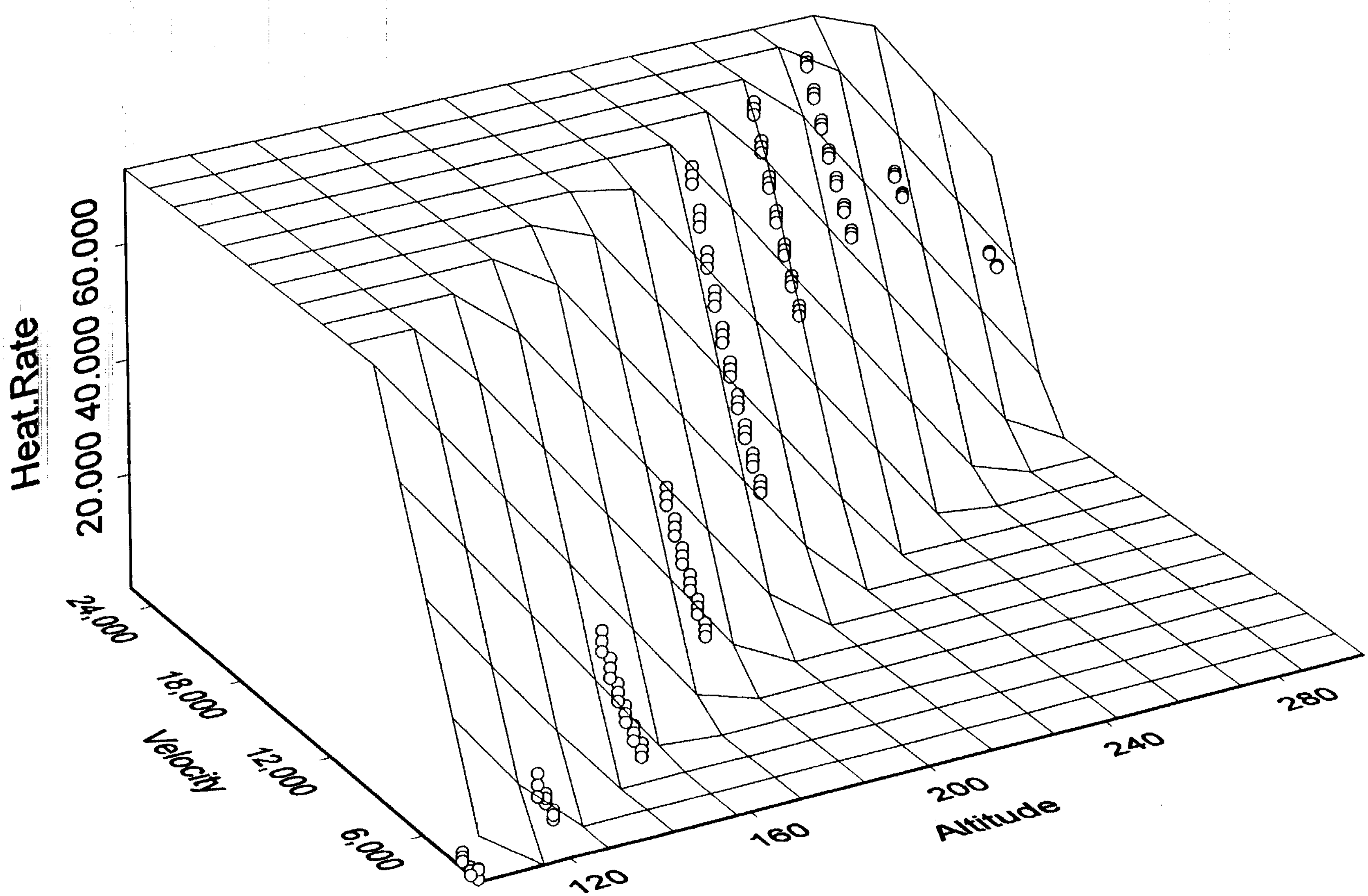

Figure 6. Contour Plot

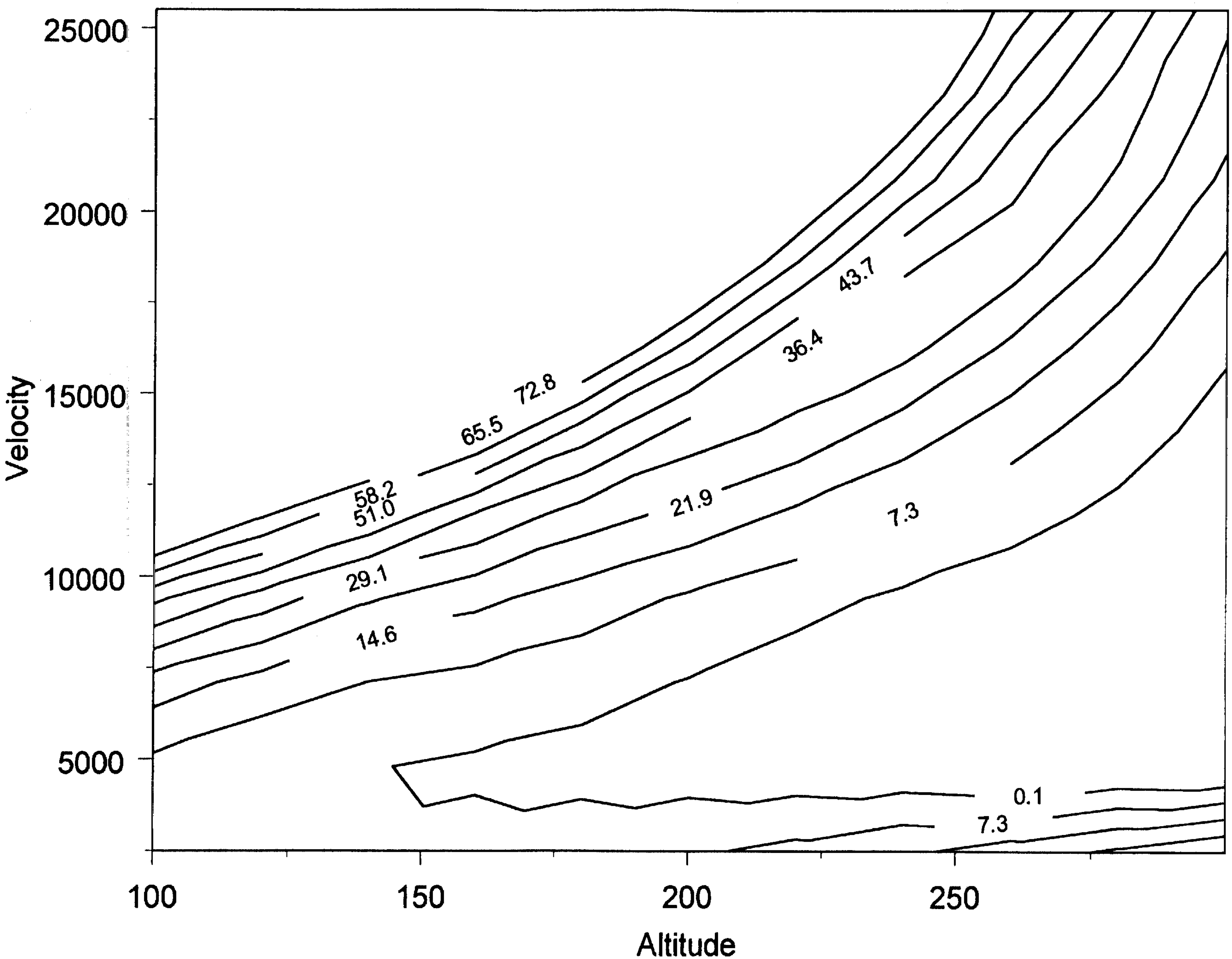

Table 11: Regression Analysis for Heat Rate (5V Corridor Data)

The regression equation is

Heat Rate, BTU/ft2/s = - 815 - 2.71 Altitude, kft
+ 0.0521 Velocity, ft/sec + 3.59 Temp, deg R
- 1.45 Dyn. Pres. lb/ft2 +0.000648 Reynolds per ft + 0.00774 Alt*2
- 0.00314 Temp*2 -0.000000 Reyn*2 -0.000101 Alt*Vel
-0.000052 Vel*Temp +0.000055 Vel*DyP +0.000283 Temp*DyP
+0.000002 DyP*Reyn

Predictor	**Coef**	**SE Coef**	**T**	**P**
Constant	-814.9	187.1	-4.36	0.000
Altitude	-2.7076	0.4786	-5.66	0.000
Velocity	0.052053	0.005991	8.69	0.000
Temp,	3.5875	0.5892	6.09	0.000
Dyn. Pre	-1.4525	0.3884	-3.74	0.000
Reynolds	0.0006481	0.0001496	4.33	0.000
Alt*2	0.007743	0.001294	5.98	0.000
Temp*2	-0.0031392	0.0005719	-5.49	0.000
Reyn*2	-0.00000000	0.00000000	-5.65	0.000
Alt*Vel	-0.00010130	0.00001424	-7.12	0.000
Vel*Temp	-0.00005249	0.00000845	-6.21	0.000
Vel*DyP	0.00005516	0.00000750	7.36	0.000
Temp*DyP	0.0002828	0.0006029	0.47	0.640
DyP*Reyn	0.00000222	0.00000035	6.42	0.000

S = 1.130 **R-Sq = 99.8%** **R-Sq(adj) = 99.7%**

Table 12: Analysis of Variance (5V)

Source	DF	SS	MS	F	P
Regression	13	68743.7	5288.0	4140.87	0.000
Residual Error	124	158.4	1.3		
Total	137	68902.1			

Table 13: Unusual Observations (5V Corridor)

Obs	Altitude	Heat Rat	Fit	SE Fit	Residual	St Resid
28	200	71.7600	69.1998	0.4198	2.5602	2.44R
41	125	1.6990	-0.5122	0.4672	2.2112	2.15R
46	100	2.8830	1.9985	0.6450	0.8845	0.95 X
92	100	2.0850	1.9985	0.6450	0.0865	0.09 X
135	125	1.7290	5.1665	0.5468	-3.4375	-3.48R
138	100	1.2880	1.9985	0.6450	-0.7105	-0.77 X

R denotes an observation with a large standardized residual

X denotes an observation whose X value gives it large influence.

Figure 7. Histogram of Residuals (5 Variables)

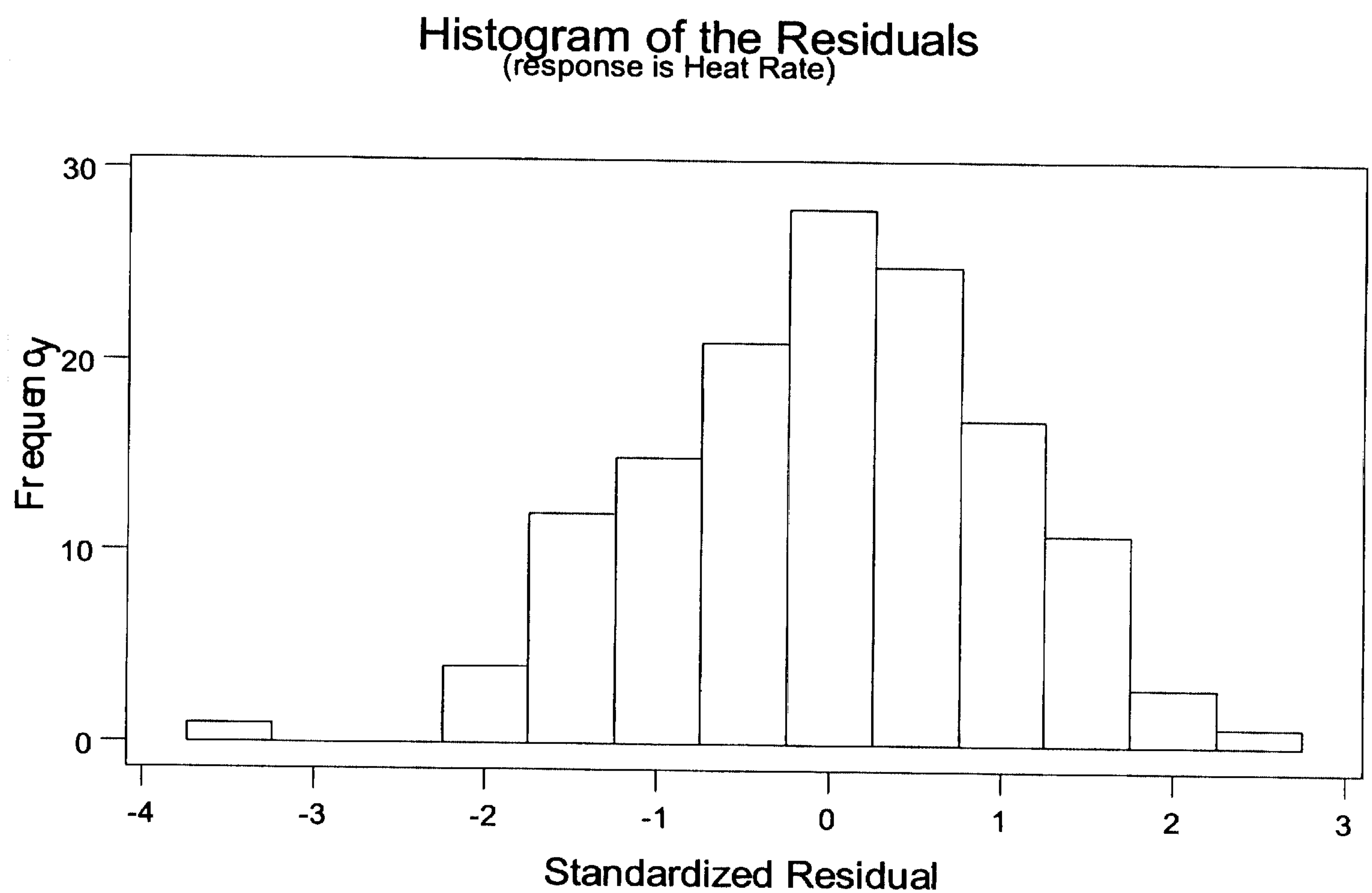

Figure 8. Normal Probability Plot (5V)

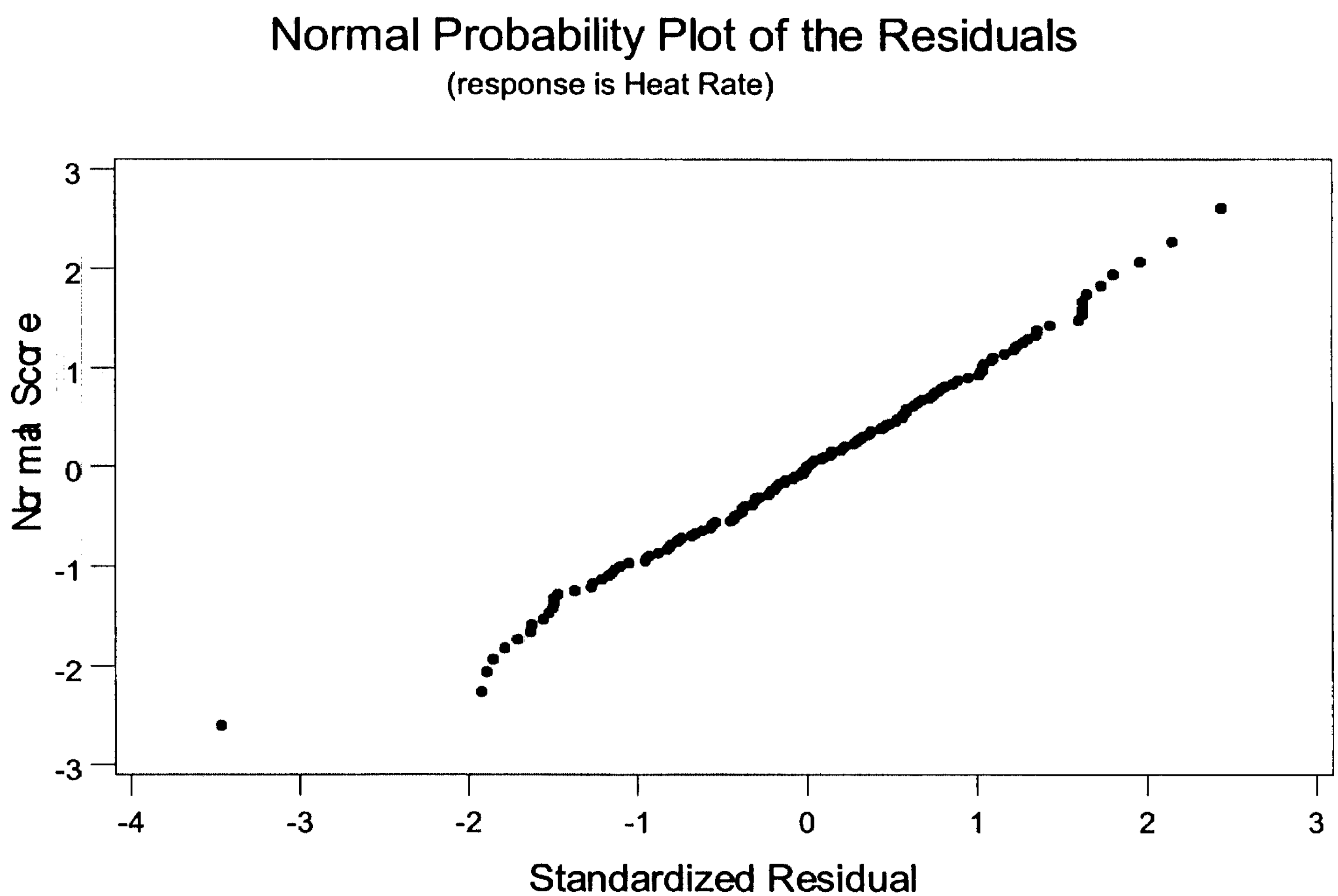

Figure 9. Residuals VS Fitted Values (5V)

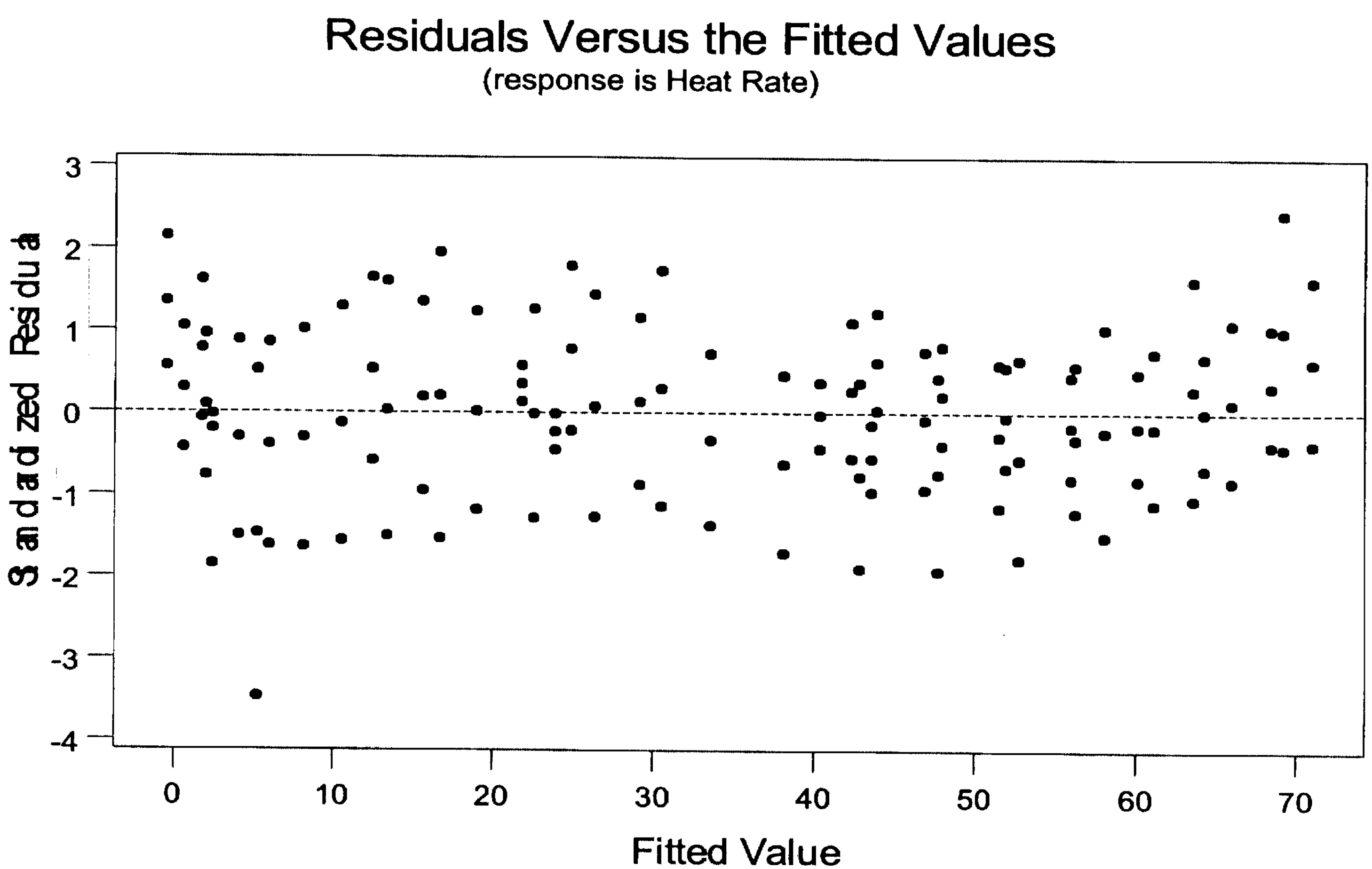

Table 14: Regression Analysis Without Alt., Vel & Twall

The regression equation is

Heat Rate, BTU/ft2/s = 241 - 0.0266 Mach*3 + 1.68 Mach*2 - 33.2 Mach

-0.000030 DynPress*3 + 0.0133 DynPress*2 - 2.99 Dyn. Pres lb/ft2

-0.000000 Energy*2 +0.000117 Energy, ft3/sec

- 43.5 Pressure, lb/ft2 +26874067 Density, slugs/ft3

- 0.0589 Temp, deg R +0.000453 Reynolds per ft

Predictor	Coef	SE Coef	T	P
Constant	241.19	61.32	3.93	0.000
Mach*3	-0.026629	0.004115	-6.47	0.000
Mach*2	1.6772	0.2470	6.79	0.000
Mach	-33.223	4.995	-6.65	0.000
DynPress	-0.00002972	0.00000566	-5.25	0.000
DynPress	0.013323	0.002475	5.38	0.000
Dyn. Pre	-2.9908	0.5574	-5.37	0.000
Energy*2	-0.00000000	0.00000000	-12.94	0.000
Energy,	0.00011696	0.00001058	11.06	0.000
Pressure	-43.51	23.52	-1.85	0.067
Density,	26874067	16016537	1.68	0.096
Temp,	-0.05891	0.08058	-0.73	0.466
Reynolds	0.0004529	0.0001233	3.67	0.000

S = 1.851 **R-Sq = 99.4%** **R-Sq(adj) = 99.3%**

Table 15: Analysis of Variance (7V)

Source	DF	SS	MS	F	P
Regression	12	68473.7	5706.1	1665.11	0.000
Residual Error	125	428.4	3.4		
Total	137	68902.1			

Table 16: Unusual Observations (7V)

Obs	Mach*3	Heat Rat	Fit	SE Fit	Residual	St Resid
1	21485	22.370	26.624	0.585	-4.254	-2.42R
4	20774	43.400	39.274	0.491	4.126	2.31R
28	4930	71.760	67.843	0.764	3.917	2.32R
44	16	0.601	1.321	1.041	-0.720	-0.47 X
46	44	2.883	1.344	1.016	1.539	0.99 X
47	21485	22.150	26.624	0.585	-4.474	-2.55R
50	20774	42.970	39.274	0.491	3.696	2.07R
90	16	2.337	1.321	1.041	1.016	0.66 X
92	44	2.085	1.344	1.016	0.741	0.48 X
93	21485	21.930	26.624	0.585	-4.694	-2.67R
109	8415	65.050	68.712	0.389	-3.662	-2.02R
110	9064	70.580	74.609	0.478	-4.029	-2.25R
136	16	2.179	1.321	1.041	0.858	0.56 X
138	44	1.288	1.344	1.016	-0.056	-0.04 X

R denotes an observation with a large standardized residual

X denotes an observation whose X value gives it large influence.

Figure 10. Histogram of Residuals (7V)

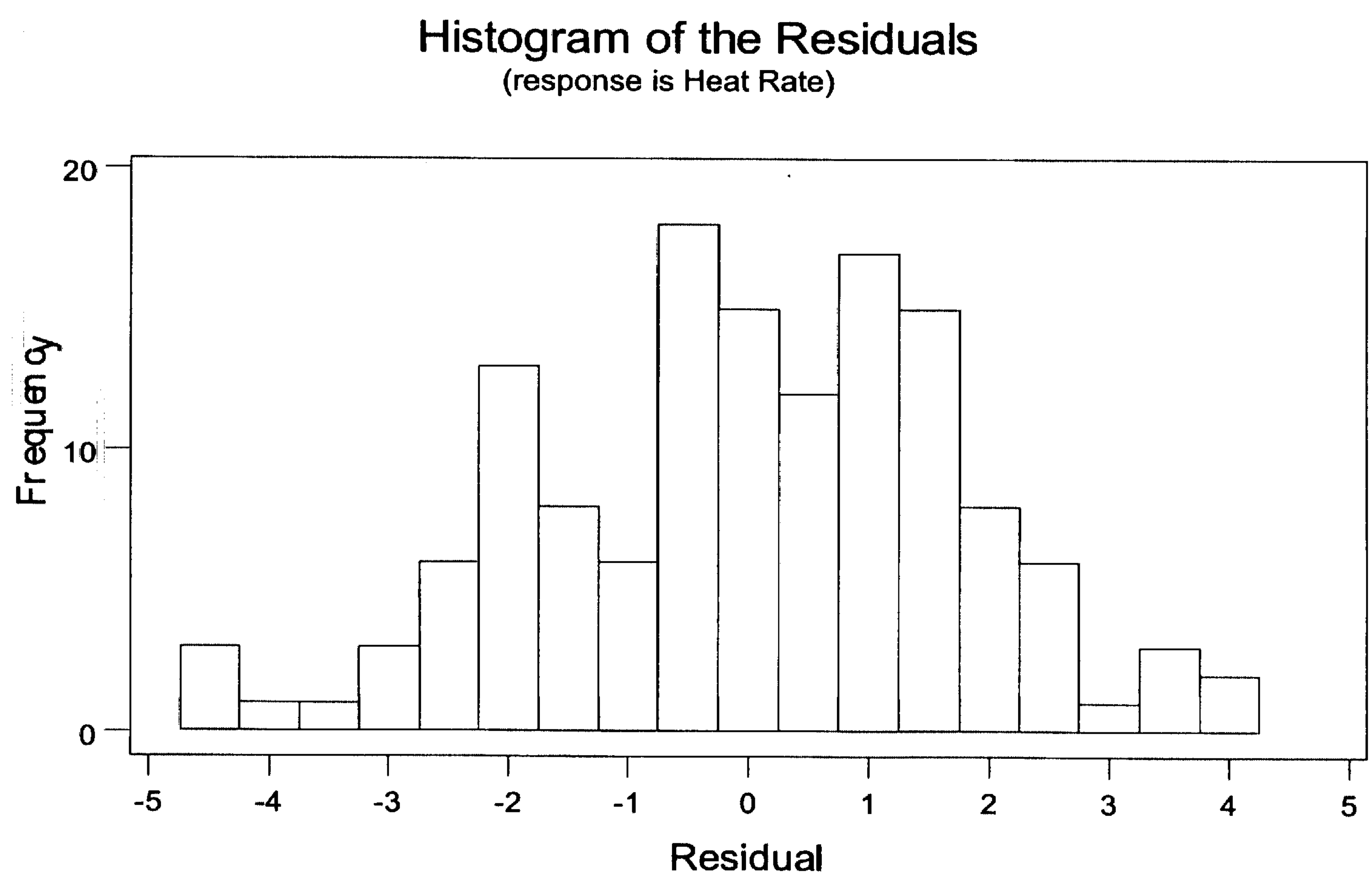

Figure 11. Normal Probability Plot (7V)

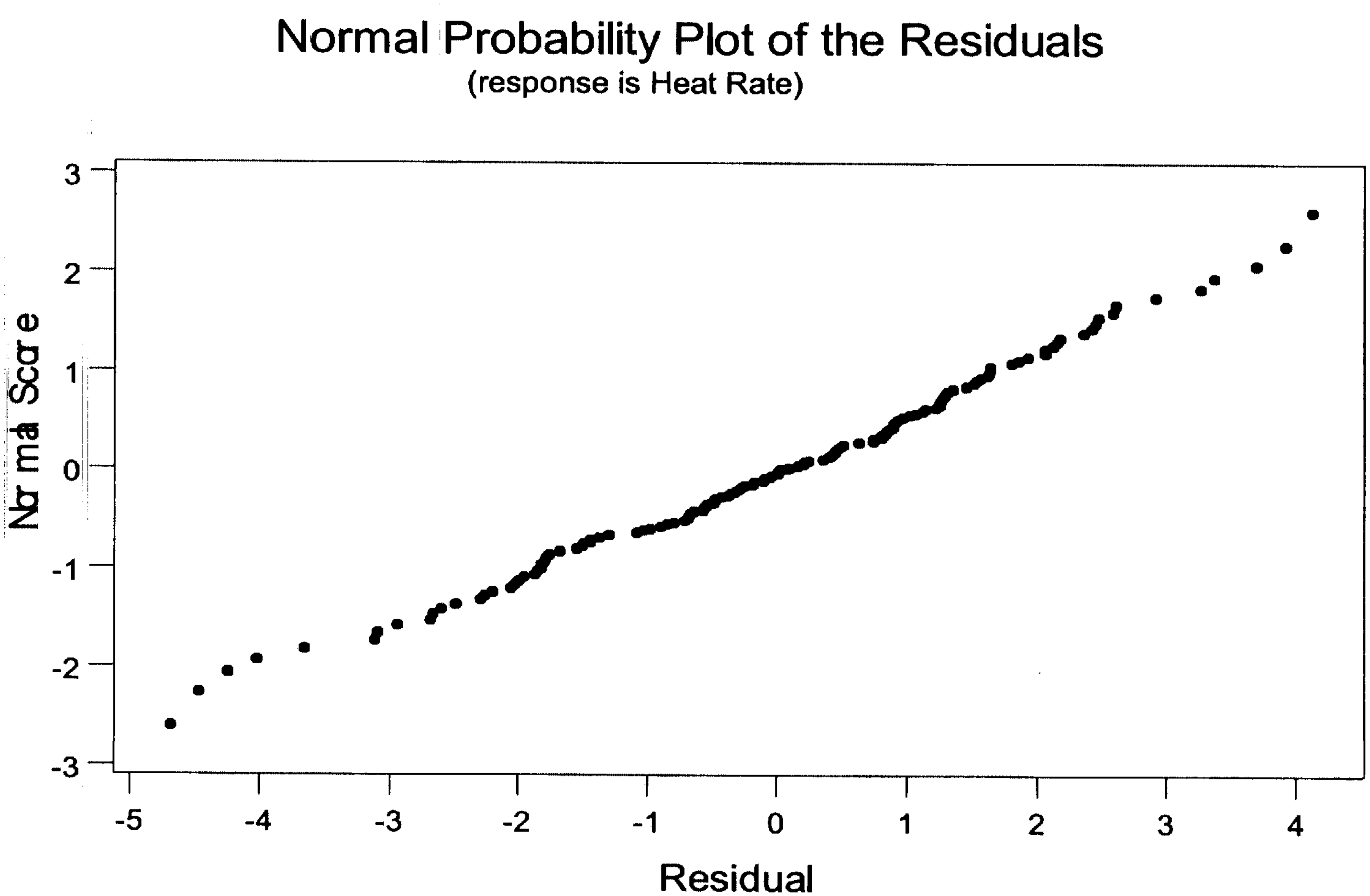

Figure 12. Residuals VS Fitted Values (7V)